Temperature dependence of universal fluctuations in the two-dimensional harmonic *XY* model

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We compute exact analytical expressions for the skewness and kurtosis in the two-dimensional harmonic *XY* model. These quantities correspond to the third and fourth normalized moments of the probability density function (PDF) of the magnetization of the model. From their behavior, we conclude that they depend explicitly on the system temperature even in the thermodynamic limit, and hence the PDF itself must depend on it. Our results correct the hypothesis called universal fluctuations, they confirm and extend previous results which showed a T dependence of the PDF, including perturbative expansions within the *XY* model up to first order in temperature.

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I. INTRODUCTION

As was originally proposed in [1] "universal fluctuations" should link very dissimilar systems, such as a confined turbulent flow and a magnetic model, at a critical point. The proposed universality was based on the probability density functions (PDFs) for the two following global quantities: The power consumption in the turbulent experiment and the magnetization in the critical system. These distributions, conveniently normalized, fell onto a common "universal" curve, which was argued to be independent of the Reynolds number and of the system size respectively. This led the authors to suggest that the universality observed in the turbulence experiment of [2] can be explained in terms of a self-similar structure of fluctuations, just as in a finite critical system. They also conjectured that this analogy should provide an important new experimental application of finite-size scaling approaches to a critical point. An extension of the observed phenomenon, which was called universal fluctuations, was made in [3] by including a wide class of quite different equilibrium and nonequilibrium systems, such as a coupled rotor model, the Ising and percolation models, models of forest fires, and sand piles among others. In all of these systems, the PDF of the corresponding global quantity when normalized to its first two moments was suggested to be of the same functional: The one corresponding to the PDF of the magnetization for the two-dimensional (2D) *XY* model in the low-temperature phase. The authors proposed that this "generalized universality" should be a consequence of systems sharing the properties of finite size, strong correlations, and self-similarity. This suggestion went far beyond the known picture of universality classes in the precise context of Wilson's renormalization group, which depends on the dimension of the system and the symmetry properties of the order parameter.

In Ref. [4] the PDF of the 2D *XY* model, in the presence of an small magnetic field, was computed by a self-consistent Hartree approximation. This small magnetic field introduces a finite but large correlation length. The authors find that the PDFs are described to an excellent approximation by a family of Gumbel distributions with exponents a, which depend on the magnetic field, and hence contain system information. Nevertheless, they state—based on earlier works [5]—that in the zero-field limit—or infinite correlation length—the PDF becomes a "universal function," independent not only of system size but also the temperature and, therefore, the critical exponent η throughout the low-temperature regime. This belief led them to compute the PDF starting with their Eq. (12), which was previously developed in the zero-magnetic field case, just by inserting the modified Green's function given by their Eq. (5) into Eq. (12). Nevertheless, this key expression for the PDF is not exact, it corresponds only to the one-loop approximation of the exact expression obtain in [17] [see Eq. (2)]. Moreover, in the same reference next to leading order corrections to the PDF are computed and it is shown that they explicity depend on the system temperature. Therefore, the one-loop approximation which is used in [4] to compute contributions to the PDF due to a small magnetic field is not trustworthy. It neglects temperature dependent corrections (see discussion below). The claim of universal fluctuations was not free of controversy since, in the comment [6], a dependence of the PDF on the universality classes was shown, using numerical data for the Ising model in two and three dimensions and for different values of the system size and temperature. Also, in [7], it was pointed out that the standard scaling form is a sufficient condition for the observed data collapse. The observations that the PDF for turbulent power fluctuations in closed flows is the same as for the harmonic 2D *XY* model is explained in these terms; nevertheless, they found significant deviations in the PDFs between the above-mentioned models when the whole 2D *XY* model is considered. The finite-size scaling of the roughness of signals in systems displaying Gaussian $1/f$ power spectra was studied in [8]. The authors found that one of the extreme value distributions (such as the Gumbel distribution proposed as the universal PDF curve) emerges as the scaling function when boundary conditions are periodic. For the case of nonperiodic boundary conditions, there are small deviations that are mainly concentrated around the maximum of the function. Although the existence of the universal BHP PDF for quite different systems was originally inferred due to the properties of finite size, strong correla-

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tions, and self-similarity that these systems shared, in the comment [9], the authors showed that the universal BHP curve can result from strong correlated data, but it need not. In this sense, the BHP curve would not be a result of some underlying critical behavior of the involved systems. The functional form used to express the universal PDF was a Gumbel distribution with exponent $a \approx \pi/2$ [3]. Nevertheless, in [10], it is shown that for a realistic range of data from different systems, the various extremal distributions when normalized to the first two moments are difficult to distinguish and, moreover, that a particular numerical value of $a \in [1,2]$ could simply arise due to a slow convergence to the Gumbel asymptote. In [11], numerical evidence for a weak but systematic temperature dependence of the PDF for the 2D *XY* model throughout the low-temperature region was initially reported. This was made by performing an accurate Monte Carlo study of the finite 2D *XY* model. This result contradicted some of the above-mentioned articles supporting the claim of universal fluctuations, and maybe because of the lack of an explicit analytical proof of the dependence of the PDF on the temperature for this model, this result remained only indirectly cited [12]. Using a stochastic cascade model of turbulence—the KO62 model—the experimentally observed non-Gaussian power fluctuations in closed turbulence was studied in [13]. The authors concluded that the asymmetric distribution found for this model strongly resembled the experimental data. Nevertheless, a very small dependence of the tails of the distributions on the Reynolds number is observed (see Fig. 2) for Reynolds numbers varying from 10^4 to 10^7. They finally argued that there was no evidence that a Gaussian distribution comes out in the limit of infinite Reynolds numbers, but given the uncertainty in the experimental results and the crudeness of the model, the qualitative agreement seemed extremely encouraging. Recently, and starting from the scaling ansatz, which is a basic assumption of the turbulence model, Chapman *et al.* [14] obtained analytically the functional form of the Gumbel PDF for such systems and found the dependence of the exponent a on system properties such as the Reynolds number in turbulence models. Furthermore, the authors pointed out, that the skewness and kurtosis are quantities sensitive to a. They also consider as a particular case of interest the 2D *XY* model in the spin-wave approximation, and starting from the previous analytical expression for the PDF of the magnetization as the Fourier transform of a sum over its moments [15], they obtained an explicit expression in the thermodynamical limit for the exponent a, which is consistent with the reported values of a: $a \approx \pi/2$ of [3] and $a \approx 1.7428$ of [5]. Nevertheless, the method used to derive the PDF is not sensitive enough to predict a direct temperature dependence of the PDF, which goes beyond the guessed Gumbel functional form parametrized by an exponent a. The exact analytical expression for the PDF for the low temperature regime of the 2D *XY* model can be found in [17] Eq. (2). More recently and by exhibiting a graph contributing to σ, the second moment of the PDF, which is not surpressed by a volume factor N and depends on the temperature T, [16], Banks and Bramwell showed that not all multiloop graphs are surpressed by factors $1/N$ and therefore may not be neglected in the thermodynamic limit, contrary to previous assumptions [5]. Using a Monte Carlo simulation, they computed the skewness s or normalized third moment of the PDF, as it "provides a clear measure of the variation of the PFD with temperature," and using a least-squares fit of the numerical data obtained, they found a numerically approximated expression for s, which they called $\gamma_3(T)$, valid for a square lattice of lattice size $L=16$,

$$s(T) \approx -0.85 + 0.126T - 0.0048T^2.$$

They also obtained for the lattice size $L=32$ "a value much closer to the theory" with $s(T) \approx -0.88 + 0.15T$, and finally argued that the skewness is relatively computational expensive due to the need for averaging and, as their results appear to confirm the evolution of s with T, they "…leave the determination of the precise form of $s(T)$ from larger systems to another time." Unfortunately, they do not only report numerical results for the skewness with numerical errors compared with the exact analytical results reported in this paper (in fact, we will find that the coefficient of T in the value of $s(T)$ for the lattice size $L=16$ is off by 7%), but its true value [see Eq. (2.7) below] decreases with the system size to an asymptotic value, contrary to their numerical results. Because of these numerical errors, it is legitimate to question the accuracy of their results and conclusions.

Recently, in [17], an analytical expression for the PDF for the full 2D *XY* model was computed systematically by means of the loop expansion, and the validity of the generalized universality was linked to renormalization group properties. The 2-loop analytical expression for the PDF shows an explicit temperature dependence. As a consequence, its skewness and kurtosis computed perturbatively up to 2-loops (first order in T) also show an explicit temperature dependence.

The aim of this paper is to deduce exact analytical expressions for the skewness and kurtosis of the PDF for the harmonic approximation of the *XY* model, defined on a square lattice of lattice size L provided with periodic boundary conditions, valid for each lattice size L and system temperature T, and from their behavior confirm that the so-called universal fluctuations depend on the system temperature. With the analytical expressions for the skewness and kurtosis, we analyze the numerical expressions and conclusions of [16] and compare them with the corresponding perturbative expressions deduced in [17] in the low-temperature region where they should be valid.

II. COMPUTATION OF THE SKEWNESS AND KURTOSIS

In this section, we deduce a general expression for the higher moments $\langle M^p \rangle$ of the PDF, valid for an arbitrary system size and to all orders in T, the system temperature. The model we are considering is the 2D *XY* model of planar spins $\phi_{\mathbf{x}}$ on a periodic 2D square lattice Λ of lattice size L, $\mathbf{x} \in \Lambda$, with nearest-neighbor cosine interactions. According to RG arguments, in the low-temperature phase and sufficiently below the Berezinskii-Kosterlitz-Thouless critical temperature the physics of this model is entirely described by its harmonic approximation, the 2D harmonic *XY* model—or Gaussian model in the language of high-energy physicists. In the present paper, exact expressions for the Gaussian model with periodic boundary conditions for arbitrary temperature and system size are obtained. Hasenbush pointed out [18] that in spite of the fact that there are no vortex contributions below the transition temperature a better approximation to the full *XY* model is obtained by taking into account the periodicity of the *XY* model for the boundary conditions. This leads to the contributions coming from the winding configurations. But they turn out to be numerically small [18] and therefore one expects that the Gaussian model with periodic boundary conditions provides a reasonable numerical approximation of the large-L limit of the *XY* model in the low-T phase. Its Hamiltonian, up to a constant, is

$$H(\phi) = \frac{1}{2} J\langle \phi, -\Delta\phi\rangle,$$

where Δ is the lattice Laplace operator, J is the ferromagnetic constant, and $\langle \phi, \varphi\rangle = \Sigma\phi(\mathbf{x})^*\varphi(\mathbf{x})$ stands for the scalar product on the lattice. We use a system of units with Boltzmann's constant equal to unity throughout, and identify T with the reduced temperature T/J. Although this model has no phase transition, it is a critical model in the sense that it has an infinite correlation length. Because this is the Hamiltonian of the Gaussian model, we use the known expression for the generating function of the Gaussian measure $d\mu_C(\phi)$ with covariance C and mean zero [19]:

$$\int d\mu_{TG}(\phi)\exp(i\langle\phi,f\rangle) = \exp\left(-\frac{1}{2}\langle f, TGf\rangle\right),$$

where the covariance in our case corresponds to $C=TG$. G denotes the lattice propagator given below by Eq. (2.6). Expanding both sides of the above equation, one obtains a closed expression for the moments $\langle M^p\rangle$ as a trace over lattice points $\mathbf{x}_l \in \Lambda$ and values $\alpha_l = \pm 1$:

$$\langle M^p\rangle = \left\{\frac{\langle M\rangle}{2N}\right\}^p \sum_{\{\mathbf{x},\alpha\}} \left\{\prod_{i\neq j} \exp\left(-\frac{1}{2}\alpha_i\alpha_j TG(\mathbf{x}_i - \mathbf{x}_j)\right)\right\}. \tag{2.1}$$

Starting from this equation and using the translational invariance of the lattice propagator G, the exact expression for the moments to all orders in T can be obtained, yielding

$$\langle M\rangle = \exp[-TG(0)/2], \tag{2.2}$$

$$\langle M^2\rangle = \frac{\langle M\rangle^2}{N}\sum_{\mathbf{x}\epsilon\Lambda}\cosh[TG(\mathbf{x})], \tag{2.3}$$

$$\langle M^3\rangle = \frac{\langle M\rangle^3}{2N^2}\sum_{\mathbf{x},\mathbf{y}\epsilon\Lambda}\left(\exp[-TG(\mathbf{x})]\cosh\{T[G(\mathbf{y}) + G(\mathbf{x}-\mathbf{y})]\} + \exp[TG(\mathbf{x})]\cosh\{T[G(\mathbf{y}) - G(\mathbf{x}-\mathbf{y})]\}\right), \tag{2.4}$$

$$\langle M^4\rangle = \frac{\langle M\rangle^4}{4N^3}\sum_{\mathbf{x},\mathbf{y},\mathbf{z}\epsilon\Lambda}\left(\exp\{-T[G(\mathbf{x}) + G(\mathbf{z}-\mathbf{y})]\}\cosh\{T[G(\mathbf{y}) + G(\mathbf{z}) + G(\mathbf{x}-\mathbf{y}) + G(\mathbf{x}-\mathbf{z})]\} + \exp\{T[G(\mathbf{x}) + G(\mathbf{z}-\mathbf{y})]\}\cosh\{T[-G(\mathbf{y}) + G(\mathbf{z}) + G(\mathbf{x}-\mathbf{y}) - G(\mathbf{x}-\mathbf{z})]\} + \exp\{T[-G(\mathbf{x}) + G(\mathbf{z}-\mathbf{y})]\}\cosh\{T[G(\mathbf{y}) - G(\mathbf{z}) + G(\mathbf{x}-\mathbf{y}) - G(\mathbf{x}-\mathbf{z})]\} + \exp\{T[G(\mathbf{x}) - G(\mathbf{z}-\mathbf{y})]\}\cosh\{T[G(\mathbf{y}) + G(\mathbf{z}) - G(\mathbf{x}-\mathbf{y}) - G(\mathbf{x}-\mathbf{z})]\}\right), \tag{2.5}$$

where Λ denotes the lattice, $N=L^2$ is the volume, and the lattice propagator G is given for example by its Fourier representation

$$G(\mathbf{x}) = \frac{1}{N}\sum_{(\mathbf{K}_L)^2\neq 0}\frac{\exp(-i\mathbf{K}\cdot\mathbf{x})}{(\mathbf{K}_L)^2}, \tag{2.6}$$

where $\mathbf{K}_L$ is the lattice momentum defined as usual as $(K_L)_i = 2\sin(K_i/2)$, with $i=1,2$, and K_i lies in the first Brillouin zone, $K_i = (2\pi/L)n$ with $n\in\mathbb{Z}$ and $-\pi < K_i \leqslant \pi$. The sum runs over all possible values of K_i for which $(\mathbf{K}_L)^2$ does not vanish. This technical point follows from the invariance of the original Hamiltonian under a global rotation of the spin variables. We now use the definitions of the skewness and kurtosis as the third and fourth normalized moments $s(T) = \langle[(M-\langle M\rangle)/\sigma]^3\rangle$ and $c(T) = \langle[(M-\langle M\rangle)/\sigma]^4\rangle$, respectively, and write

$$s(T) = \frac{1}{\{\langle M^2\rangle - \langle M\rangle^2\}^{3/2}}[\langle M^3\rangle - 3\langle M^2\rangle\langle M\rangle + 2\langle M\rangle^3], \tag{2.7}$$

$$c(T) = \frac{1}{\{\langle M^2\rangle - \langle M\rangle^2\}^2}[\langle M^4\rangle - 4\langle M^3\rangle\langle M\rangle + 6\langle M^2\rangle\langle M\rangle^2 - 3\langle M\rangle^4]. \tag{2.8}$$

Approximated analytical expressions can be obtained by expanding each term in powers of T, obtaining up to first order in T

$$s(T) = -g_3\left(\frac{2}{g_2}\right)^{3/2}\left\{1 - \frac{3}{4}\frac{(g_2)^2}{g_3}T + O(T^2)\right\}, \tag{2.9}$$

$$c(T) = 3\left\{1 + \frac{4g_4}{(g_2)^2} - \frac{8g_3}{g_2}T + O(T^2)\right\}, \tag{2.10}$$

where the quantities g_n are defined in terms of the power n of the lattice propagator G as $g_n = G^n(0)/N^{n-1}$. The lattice coefficients, g_n for $n\geqslant 2$, depend weakly on the volume N in the thermodynamic limit. From the above equations, we observe that the linear coefficients—which we call slopes—appearing in the temperature expansions of the skewness and kurtosis are expressed in terms of the g_n with $n\geqslant 2$. These coefficients have a nontrivial thermodynamic limit. Numerically, we found that in the large-L limit they do not vanish and get the values 0.1319 and −0.470, respectively. This fact shows clearly that the first high-order moments of the PDF depend on the system temperature even in the thermodynamic limit, and hence the PDF itself depends explicity on it. Here, I want to point out that Eqs. (2.9) and (2.10) can be obtained directly from [17] as follows: in Eqs. (31) and (34) of [17], the skewness and kurtosis were computed up to 2-loops (or equivalently up to first order in T) including anharmonic corrections. But according to their conclusions, these corrections merely contribute to a renormalization of temperature, up to order N^{-1}, $T \to T(1+\lambda T+\cdots)$. Due to the fact that the harmonic corrections do not include terms supressed by factors $1/N$, we conclude that up to first order in the temperature, Eqs. (2.9) and (2.10) can be obtained from Eqs. (31) and (34) of Ref. [17] by neglecting the $1/N$ terms. These contributions are negligible in the thermodynamic limit but are relevant for finite lattice sizes, as we will show numerically in the next section. Higher-order corrections in T can be directly computed along the lines outlined here.

III. NUMERICAL RESULTS

In order to compare with values reported in [16], we evaluate numerically Eqs. (2.9) and (2.10) by using MATLAB for different lattice sizes including $L=16$ and $L=32$. We obtain for the skewness and kurtosis the approximate expressions $s(T) \approx -0.8540+0.1358T$, $c(T) \approx 4.3283-0.4639T$ for $L=16$, and $s(T) \approx -0.8763+0.1331T$, $c(T) \approx 4.3820 -0.4666T$ for $L=32$ respectively. Independent of the numerical differences found for the first two coefficients of the skewness, one observes that the values for the slope (linear term in T), which we will denote by $m(L)$, reported in [16] increase with the system size, [they report the values $m(16)=0.126$ and $m(32)=0.15$ for the slope, in contrast with ours $m(16)=0.1358$ and $m(32)=0.1331$], contrary to the scale behavior of the coefficients appearing in Eq. (2.9). Moreover, we observe that the difference between the values obtained for the slope in [16] and in this paper grows with the system size from 7% to 11% for the lattice sizes $L=16$ and 32. From its analytical expression given in (2.9), it turns out that the slope is a decreasing function of the system size and its thermodynamic limit converges to the value 0.1319. From the expression for the kurtosis given by Eq. (2.10), we can obtain also the large-L limit value of the slope which gives -0.470. Finally, and in order to compare our results with the accurate but perturbative values obtained in [17] for the skewness and kurtosis, we evaluate numerically the contributions obtained from the anharmonic corrections, which are, according to the discussion of the last paragraph of the previous section, the $1/N$-terms:

$$\delta s(T) = \frac{3}{2N} g_3 \left(\frac{2}{g_2}\right)^{3/2} \left(\frac{g_1^2}{2g_2} - \frac{g_1 g_2}{g_3}\right) T, \qquad (3.1)$$

$$\delta c(T) = \frac{1}{N}\left(2g_1 g_3 + g_2^2 - \frac{g_1^2 g_4}{g_2}\right)\frac{12}{(g_2)^2} T. \qquad (3.2)$$

One obtains $\delta s(T) \approx 0.0197T$, $\delta c(T) \approx -0.0404T$ for $L=16$ and $\delta s(T) \approx 0.0191T$, $\delta c(T) \approx -0.0423T$ for $L=32$, respectively. Perfect agreement is found with the corresponding values of [17] [see Eqs.(36) and (38)], when adding these corrections to the values obtained in Eqs. (2.9) and (2.10) for $s(T)$ and $c(T)$, respectively.

This remarkable agreement represents an indirect test of the RG-argument given in [17], that the effect of the corrections to the spin-wave approximation (anharmonic corrections) on the PDF is merely a renormalization of temperature, up to order $1/N$.

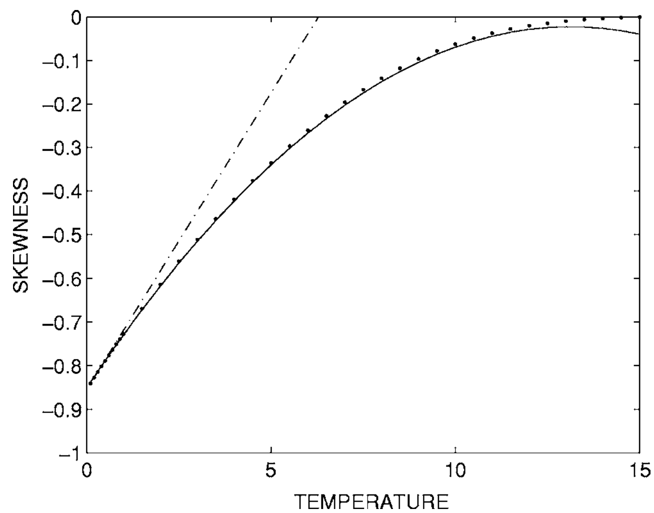

FIG. 1. The skewness is plotted as a function of T/J in the range $[0,15]$, for a lattice of lattice size $L=16$. The dotted-dashed line corresponds to its linear approximation given by Eq. (2.9), the dotted line represents the exact expression given by Eq. (2.7), and the full line corresponds to the quadratic fit of Monte Carlo data found in [11] [see Eq. (20)].

To show the temperature dependence of these quantities we plot in Figs. 1 and 2 the skewness and kurtosis for a square lattice of lattice size $L=16$ in the temperature range $[0,15]$. In the range of small values of temperature $T \in [0,1]$, the skewness and kurtosis show a linear dependence in agreement with their linearized expressions given by Eqs. (2.9) and (2.10). For larger values of T, the skewness grows slightly faster than the quadratic fit of Monte Carlo data found in [16] [see Eq. (20)]. The difference between both curves is a monotonically growing function of the tempera-

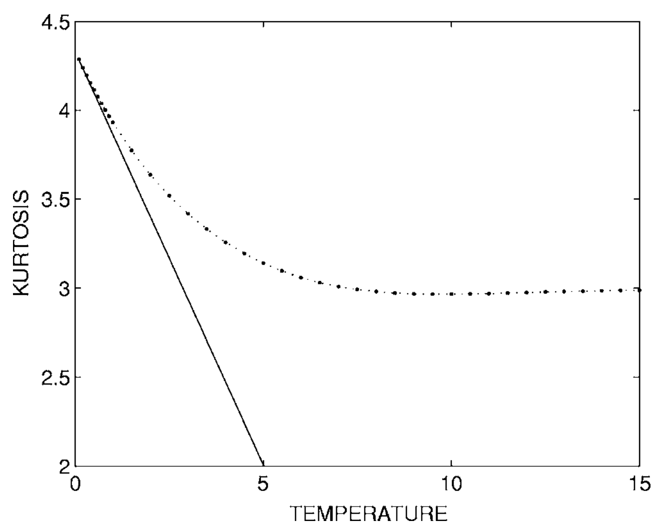

FIG. 2. The kurtosis as a function of the reduced temperature T/J for a lattice of lattice size $L=16$. The full line corresponds to its linear approximation given by Eq. (2.10), and the dotted line represents the exact expression given by Eq. (2.8).

ture. For large values of T, the kurtosis displays a weak convexity and appears to reveal oscillations toward the value $c=3$.

IV. CONCLUSIONS

From the low-temperature expansion for the skewness and kurtosis given by Eqs. (2.9) and (2.10), we observe that the linear coefficients—which we call slopes—have nontrivial thermodynamical limits. Numerically, we found that in the large-L limit they do not vanish and get the values 0.1319 and −0.470, respectively. This fact shows clearly that the first high-order moments of the PDF depend on the system temperature even in the thermodynamic limit and, hence, the PDF itself depends explicity on it, contrarily to the claim of universal fluctuations found in the literature throughout many years since its original proposal [1]. This conclusion agrees with previous numerical results [11] and with the main result of [17].

The exact expressions found for both quantities, the skewness and kurtosis, and their numerical evaluations plotted in Figs. 1 and 2, agree perfectly in the low-temperature region with the 2-loop approximated analytical expressions reported in [17] when the anharmonic corrections are taken into account, where these expansions for them should apply.

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